Origin of DC voltage in type II superconducting flux pumps: field, field rate of change, and current density dependence of resistivity

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Superconducting flux pumps are the kind of devices which can generate direct current into superconducting circuit using external magnetic field. The key point is how to induce a DC voltage across the superconducting load by AC fields. Giaever [1] pointed out flux motion in superconductors will induce a DC voltage, and demonstrated a rectifier model which depended on breaking superconductivity. Klundert et al. [2, 3] in their review(s) described various configurations for flux pumps all of which relied on inducing the normal state in at least part of the superconductor. In this letter, following their work, we reveal that a variation in the resistivity of type II superconductors is sufficient to induce a DC voltage in flux pumps and it is not necessary to break superconductivity. This variation in resistivity is due to the fact that flux flow is influenced by current density, field intensity, and field rate of change. We propose a general circuit analogy for travelling wave flux pumps, and provide a mathematical analysis to explain the DC voltage. Several existing superconducting flux pumps which rely on the use of a travelling magnetic wave can be explained using the analysis enclosed. This work can also throw light on the design and optimization of flux pumps.

1 Introduction

Under development for some years Coated Conductors (CC) have struggled to find application in high field magnet systems such as Magnetic Resonance Imaging (MRI) [4] and Nuclear Magnetic Resonance (NMR) [5]. This has chiefly stemmed from the fact that coils constructed from CC are difficult to operate in persistent mode due to the relatively low n-value [6] and the relatively high resistance in joints [7]. If a flux pump [8-18] is used then current leads and persistent current switches [19] are not required. The magnet's field can be maintained using the pump and the coil can be operated in persistent mode. The idea of using a travelling magnetic wave to gradually magnetize a type-II superconductor was firstly proposed by Coombs [8, 9]. After that, several High-T_c Superconducting (HTS) flux pumps based on travelling wave were developed for CC coils [10-16]. These flux pumps use a piece of CC (CCs) connecting to a superconducting load. When magnetic field travels across the CC, flux gradually accumulates in the load. The key point of these flux pumps is how a DC voltage is induced by external fields, which has also been confusing for years. The DC transformer which predates the Flux Pumps described by Van Klundert [2, 3] et al appears in Gieaver [1] who pointed out that flux motion can be used to induce a DC voltage in a superconductor and described a rectifier based on a superconducting switch. Following their work, we will reveal that varying resistivity of type II superconductors due to flux flow is the origin of the DC voltage and therefore flux pumping. The resistivity is influenced by current density in the superconductor, flux density experienced by the superconductor, and field rate of change. The proposed principle can well explain existing travelling wave flux pumps.

2 Basic principle

2.1 Circuit model

Travelling wave flux pumps comprise a superconducting loop connecting to a superconducting load which we want to magnetize. The superconducting loop is subjected to magnetic fields which vary in time and space, these induce a voltage across the branch which is connected to the load, as shown in Fig. 1(a). If the open circuit voltage $v(t)$ has a DC component V_{DC}, it will generate a increasing DC current in an inductive load. Therefore, the DC component is substantial in flux pumping. If the perimeter of the loop is considerably larger than the width of branches, and the resistance of this loop along its length is considerably larger than its inductance, Fig. 1(a) can be described by Fig. 1(b) as a circuit model, where the right branch of the circuit represent the branch *ab* in Fig. 1(a), the left branch in the circuit represents branch *adcb* in Fig. 1(a), $v_1(t)$ and $v_2(t)$ represent the induced EMF forces in each branches, and $R_1(t)$ and $R_2(t)$ represent resistance of the branches accordingly.

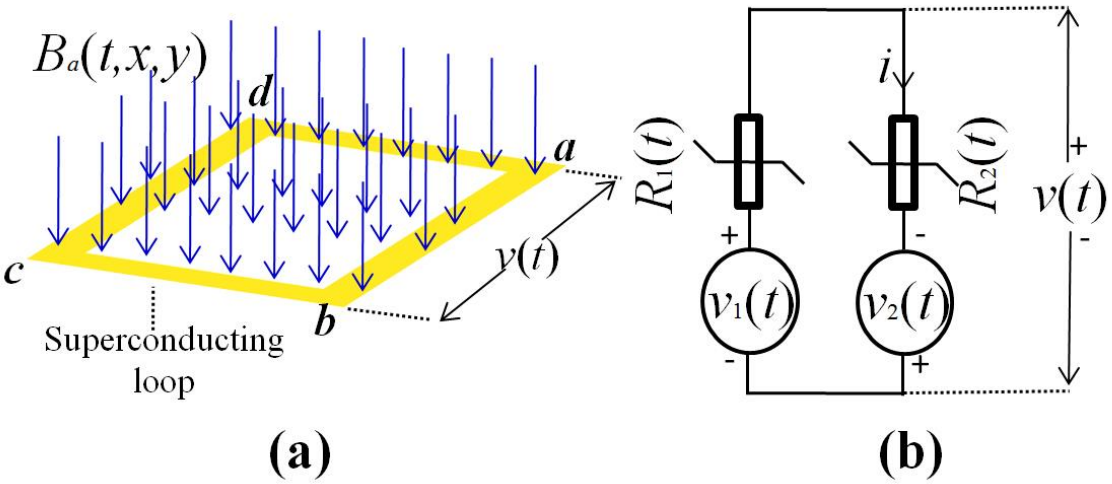

FIG. 1. Schematic drawing of open circuit voltage of travelling wave flux pump. (a) Magnetic field varying in time and space is applied to a superconducting loop, part of which will be connected to a superconducting load. (b) Circuit analogy of travelling wave flux pump, where $v_1(t)$ and $v_2(t)$ represent the induced EMF forces in each branch, and $R_1(t)$ and $R_2(t)$ represent the resistance of each of the branches.

According to Faraday's Law:

$$v_1(t)+v_2(t) = \int_{adcb} \vec{E}\cdot d\vec{l} + \int_{ba} \vec{E}\cdot d\vec{l} = \oint_l \vec{E}\cdot d\vec{l} = -\int_S \frac{d\vec{B}}{dt}\cdot \boldsymbol{n}ds = -d\Phi / dt \quad (1)$$

Where l is the perimeter of the loop, S is the area of the loop, B is the applied field, and Φ is the total flux applied to the loop.

The open circuit voltage $v(t)$ across the branch is :

$$v(t)=iR_2(t)-v_2(t)=\frac{v_1(t)+v_2(t)}{R_1(t)+R_2(t)}R_2(t)-v_2(t) \quad (2)$$

The DC component in $v(t)$ is:

$$V_{DC}=\frac{1}{T}\int_0^T v(t)dt=\frac{1}{T}\int_0^T \frac{-d\Phi/dt}{R_1(t)+R_2(t)}R_2(t)dt \quad (3)$$

Where T is the period of the applied field in the area of the loop. Here we should consider $\int_0^T v_2(t)dt=0$ (Otherwise an AC magnetic field would induce a DC electric field, which is against Faraday's Law). It should be noticed that the same conclusion can be drawn by analyzing the left branch in Fig. 1(b). **If Eq. (3) is non-zero then flux pumping can be achieved**.

In Eq. (3), if $R_2(t)/(R_1(t)+R_2(t))$ is constant, then we can get:

$$V_{DC}=\frac{1}{T}\frac{R_2(t)}{R_1(t)+R_2(t)}\int_0^T -d\Phi/dtdt=0 \quad (4)$$

However, if $R_2(t)/(R_1(t)+R_2(t))$ is not constant during Φ increasing and decreasing process, Eq. (3) may be non-zero. For simplicity, we assume that $R_2(t)/(R_1(t)+R_2(t))$ is constant during Φ increasing or Φ decreasing, and define $p_{dec}=R_2(t)/(R_1(t)+R_2(t))$ for Φ decreasing, and $p_{inc}=R_2(t)/(R_1(t)+R_2(t))$ for Φ increasing, then we can get:

$$V_{DC}=\frac{1}{T}(p_{dec}\int_0^{t_1} -d\Phi/dtdt+p_{inc}\int_{t_1}^T -d\Phi/dtdt)=\frac{1}{T}(p_{dec}-p_{inc})\Delta\Phi \quad (5)$$

Where $t=t_1$ is the time flux is minimum, $t=0$ is the time flux is maximum, and $\Delta\Phi$ is the peak-peak value of Φ applied to the loop. According to Eq. (5), if the ratio of $R_1(t)$ and $R_2(t)$ changes in the flux increasing process and flux decreasing process, a DC component in the open circuit voltage will occur. The physics of p_{dec} and p_{inc} can be understood as follows: p_{inc} describes the proportion of flux that flows into the loop via branch *ab* in Fig. 1(a); p_{dec} describes the proportion of flux that flows out of the loop via the same branch. Since the net flux variation in the loop is zero, if $p_{inc}\neq p_{dec}$, it means a net flux flows across branch *ab*. This net flux flow induces a DC voltage. This analysis also applies to branch *adcb* in Fig. 1(a).

For a superconducting loop which has a large inductance, the situation is slightly different. But the inductance only influences the relative phase between the current and the applied flux in the loop. It will not change the substance that the variation of resistance in branches generates a DC component in the open circuit voltage.

For practical application, we may want V_{DC} as large as possible. Various methods can be used to increase the value of V_{DC}. E.g. By changing the value of $R_1(t)$ and $R_2(t)$, p_{dec}-p_{inc} varies in the region of (-1,1). For example, if $R_2(t)$ is much larger than $R_1(t)$ when Φ is decreasing and $R_2(t)$ is much smaller than $R_1(t)$ when $\Phi$is increasing, then p_{dec}-$p_{inc}\approx$-1. Additionally by increasing field frequency, 1/T can be increased. Also by increasing field magnitude or area, $\Delta\Phi$ can be increased.

2.2 Influential factors on branch resistances

For a type-II superconductor, its resistivity varies with current density, field intensity and field rate of change.

For example the branch resistance depends on current density and applied field, as described in *E-J* power law [20] under Kim's Model [21]:

$$R=\int_L \rho sdL=\int_L E_0 J^{n-1}/(\frac{J_{c0}}{1+B/B_0})^n sdL \qquad (6)$$

Where ρ is the resistivity, s is the cross section of the branch, and L is the length of the branch.

Additionally, the change in applied field will generate a loss, which can be considered as an equivalent AC loss resistance [22] or dynamic resistance [14, 23-28]:

$$R_{dyn}=\frac{2afL}{I_c}(B_a\text{-}B_{a,th}) \qquad (7)$$

Furthermore, the crossed-magnetic-field effect [29] and flux cutting effect [30] can also contribute to the variation of resistance, and thus contributes to pumping.

For type II superconductor, if the geometries of the branches are the same and they are in an homogeneous AC magnetic field, $R_2(t)/(R_1(t)+R_2(t))$ is always constant, so no DC voltage can be generated. However, if the geometries are different, e.g. the branches have different current capacities, a DC voltage may occur even if the loop is in homogeneous field.

3 Explanation of travelling wave flux pumps

Several existing traveling wave flux pumps can be considered as the realization of Eq. (5) in the previous section, since in these flux pumps, an inhomogeneous AC magnetic field travels across the superconducting branches. This is explained as follows: When a travelling wave passes across the superconductor the two branches see different portions of the wave at the same instant. Two things result from this: in the first the flux density seen by each branch is different and in the second the rate of change of flux density is also different.

3.1 Flux pumping due to field dependence of branch resistances

Fig. 2(a) shows a symmetrical triangular magnetic wave traveling across a superconducting loop. The loop is formed from two branches with same geometry which are infinitely long into the page.

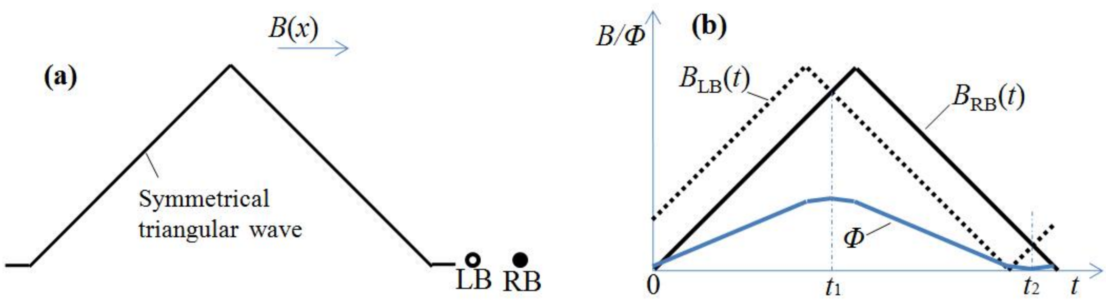

FIG. 2. Schematic drawing of a superconducting loop experiencing a symmetrical triangular travelling magnetic

wave. (a) travelling magnetic field is proceeding towards a superconducting loop, which is formed by two branches infinitely long into the paper. LB denotes left branch, and RB denotes right branch. (b) flux density experienced by the two branches against time, and total flux applied to the loop.

In Fig. 2(b), the two branches both experience a symmetrical triangular wave field and the phase difference is due to the different physical positions of each branch. During the time t=0 to t=t_1, the total flux in the loop is *increasing*, and the left branch is seeing a *higher* flux density than the right branch. During the time t=t_1 to t=t_2, the total flux in the loop is *decreasing*, and the left branch is seeing a *lower* flux density than the right branch. As is shown in Eq. (6) the branch resistances are dependent on the applied field. Thus resistance of the left branch is larger than that of the right branch in the flux increasing process, and is smaller during flux decreasing process. Therefore Eq. (5) predicts that a DC voltage will be induced.

3.2 Flux pumping due to field rate of change dependence of branch resistances

Fig. 3 shows the branches experiencing a narrow rectangular wave, in which the distance between the rise edge and the falling edge is shorter than the distance between the branches. During time t=t_1 and t=t_2 the total flux in the loop increases, and the left branch experiences a fast change in field, which generates a loss (which can be considered as a dynamic resistance since the field in left branch changes much faster than the current in the loop, otherwise it can be considered as an AC loss resistance). During time t=t_3 and t=t_4 the total flux in the loop drops, and the right branch experiences a fast change in field, which generate a loss. So the ratio of resistances in the branches changes during flux rising and falling, thus resulting in a DC component in the open circuit voltage.

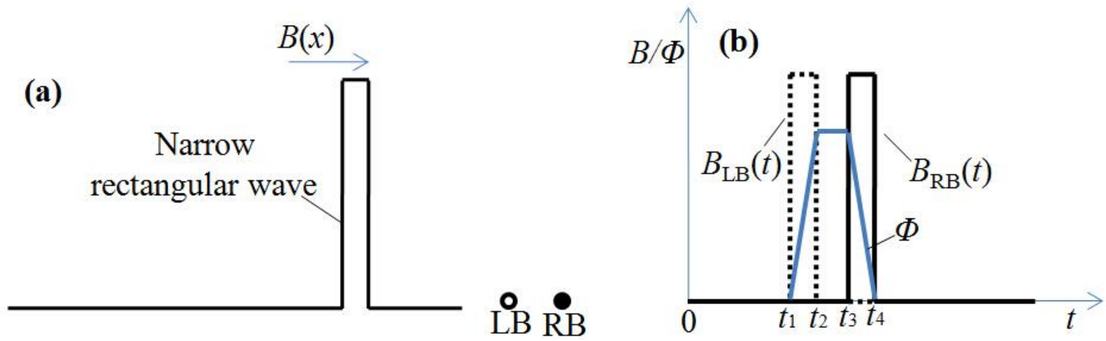

FIG. 3. Schematic drawing of a superconducting loop experiencing a narrow rectangular travelling magnetic wave. (a) travelling magnetic field is proceeding towards a superconducting loop, which is formed by two branches infinitely long into the paper. LB denotes left branch, and RB denotes right branch. (b) flux density experienced by the two branches against time, and total flux applied to the loop.

3.3 Experimental validation

We constructed a linear travelling wave based flux pump, which is shown in Fig. 4(a). Four pairs of copper poles are used to generate travelling waves of various shapes. Four parallel placed CC tapes experience the travelling magnetic wave which is orientated perpendicular to their faces. The tapes are connected to a coil constructed from CC. The set up can be considered as two separate loops working in parallel. We applied a triangular magnetic wave with different duty ratios (i.e. we varied the ratio of the rise time to the period), and the results are shown in Fig. 4(c). The current induced in the coil rises exponentially which corresponds to it being induced by a DC voltage with an internal resistance. The results presented here show that different duty ratios end up with different load currents. The difference between field rise time and fall time influences not only the final load current magnitude but also the polarity and the rate of rise.

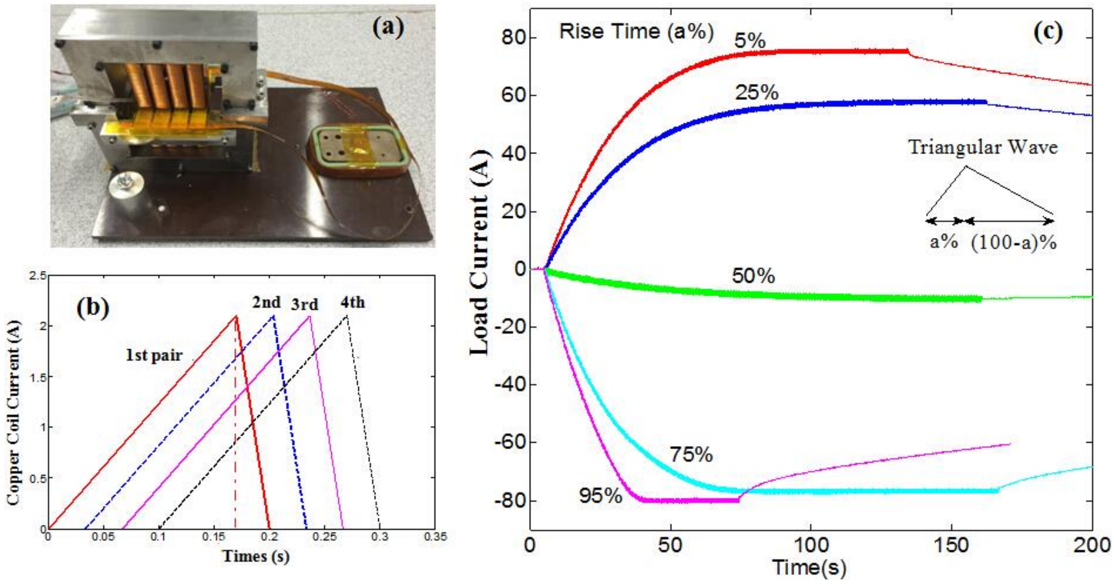

FIG. 4. Linear HTS flux pump device and flux pumping result. (a) The picture of the flux pump, which has 4 pole pairs that can generate a travelling magnetic wave. (b) The waveform of current in each pole pair. (c) The load current under different waveforms.

3.3.1 The effect of field dependence of resistance

For a symmetrical waveform, which is similar to the description in Fig. 2, a very small amount of current is pumped into the load with a low speed, which indicates a low DC voltage is induced. This result shows that the field dependence of the critical current density (branch resistance) is not the key influential factor in the experiment. This may be because the circulating current in the loops is small compared to branches critical current.

3.3.2 The effect of field rate of change dependence of resistance

When the duty ratio is changed then the field rate of change is markedly different for the two branches. The current rises more rapidly which indicates the DC voltage is higher. This is similar to the process described in Fig. 3, where the loss caused by the rate of change of the field has become the dominant factor in the experiment.

A similar result is obtained for moving magnet based flux pumps [11-15], if more than one piece of tape forms the superconducting loop, it is very similar to the description in Fig. 3. The published results [11, 13] show that the pumping speed is nearly proportional to rotating frequency, and the load current polarity is related to rotating direction, which proves our assumption. If only a single piece of tape experiences the moving magnetic field, it may also be considered as a loop as the induced currents will circulate within the tape [1]. In this case there is an additional factor as the size of the two branches is not fixed.

Although this discussion has concentrated on travelling waves, we showed in [18] that it is not necessary for a travelling wave to be present for a DC voltage as predicted by Eq. 5 to be induced. In [18] we constructed a transformer-rectifier flux pump. A transformer was used to induce EMF in the loop, and a high frequency field was intermittently applied to one branch to change its resistance, thus resulting in flux pumping.

4 Conclusion

In conclusion, we have revealed variation in the resistivity of type II superconductors is the origin of the DC component of the open circuit voltage of travelling wave flux pumps. Because the resistivity of type II superconductors depends on the field magnitude, field rate of change, and current density, when magnetic fields with different magnitudes and different changing rates in space are applied to a superconducting loop, there is different and differing resistivity around the loop. This results in a DC open circuit voltage and is the origin of flux pumping in these devices. In a type I superconducting flux pump, a normal spot is used to transport flux into a superconducting loop, whereas in a type II or travelling wave type flux pump the normal spot is not necessary. The variable resistivity property is particularly evident in High Temperature Superconductors making them ideal candidates for this type of pump.

Acknowledgements

Jianzhao Geng would like to acknowledge Cambridge Trust for offering Cambridge International Scholarship to support his study in Cambridge.

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